

# Experiment on a one-way flow of a rarefied gas through a straight circular pipe without average temperature and pressure gradients

Y. Sone, T. Fukuda, T. Hokazono, and H. Sugimoto\*

\*Department of Aeronautics and Astronautics, Graduate School of Engineering, Kyoto University, Kyoto 606-8501, Japan

**Abstract.** It is demonstrated experimentally that a one-way flow of a slightly rarefied gas is induced in a circular straight pipe of a uniform diameter with its two ends being kept at an equal temperature and pressure by heating the middle part along the pipe and by putting up a shelf or shelves inside of a half part of the pipe on one side of the heater. The experiment is done in a small vacuum chamber of a glass bell jar, and the flow is detected by a small windmill. A one-way flow is induced from the part of the pipe with a shelf or shelves to that without it or them. The mechanism of the flow is explained by the difference of dependence on the cross sectional area between the thermal transpiration and Poiseuille flow.

## INTRODUCTION

Engineering application, to such as a pumping system or flow control, of a flow induced by a temperature field in a rarefied gas, such as thermal transpiration (Knudsen [1], Kennard [2]), has been of interest for a long time because of absence of a mechanical (or moving) part, and more attention has been paid anew for it recently in relation to micromechanical system (*e.g.*, Huber [3], Pham-van-Diep [4], Sone *et al.* [5], Vargo and Muntz [6], Hudson and Bartel [7]). In order to induce a large momentum or a large pressure difference in a gas without imposing a large temperature difference on the system, some cascade system should be devised. The study of pressure difference induced between the two ends kept at an equal temperature of a closed system, which is initiated by Knudsen [8], has been carried out anew by various authors [4–7]. The counterpart problem is the study of a one-way flow in a pipe or channel with its two ends kept at an equal condition (equal temperature and equal pressure) or in an infinitely long pipe or channel system with a periodic condition along the pipe or channel. The flow and its pumping effect are studied numerically in Sone *et al.* [5] and Aoki *et al.* [9]. In the previous paper [10], the first author, in collaboration with Sato, demonstrated that a one-way flow is induced in a circular pipe with its two ends kept at an equal condition by changing the diameter in the middle where the pipe is heated. No one-way flow was induced in a pipe of a uniform diameter, even when it was heated at the portion well off the middle part. The corresponding result for a pipe or channel of infinite length is confirmed mathematically (Golse [11]) and numerically (Sone *et al.* [5], Aoki *et al.* [9]). The present paper is a demonstration of a one-way flow in a pipe of a uniform diameter but, instead, with a shelf or shelves being put up in a half part of the pipe on one side of the heater. As will be explained later, the physical mechanism of the flow is due to the difference of dependence on the cross sectional area between the thermal transpiration flow and Poiseuille flow.

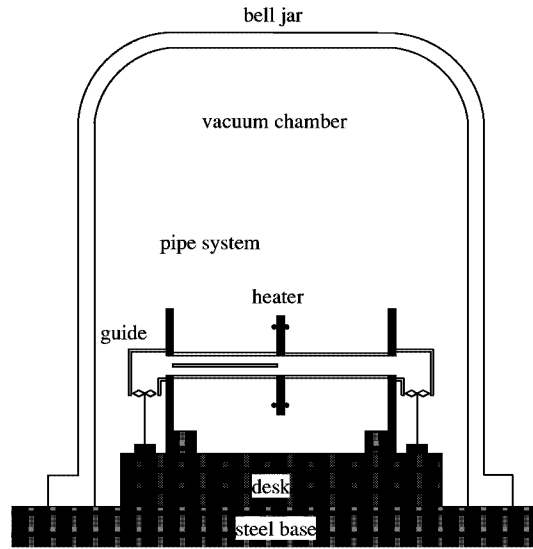
## EXPERIMENT

### Method and apparatus

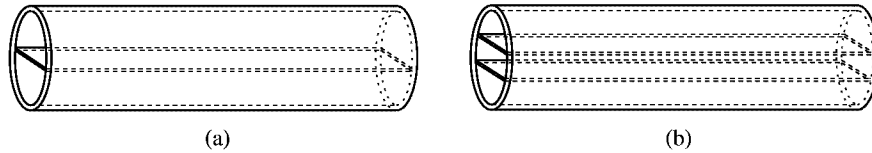
The basic plan of the experiment is as follows: a straight circular pipe of a uniform diameter with a heater at the middle position along the pipe is prepared. A shelf or shelves are put up inside a half part of the pipe on one side of the heater. Both ends of the pipe are open and are devised to be kept at an equal temperature. The pipe with the heater on is set in a vacuum chamber. Then, on each side of the heater in the pipe, a thermal transpiration flow is induced toward

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**FIGURE 1.** Experimental apparatus.



**FIGURE 2.** Shelf or shelves in a pipe. (a) Pipe 2 (pipe with a shelf) and (b) Pipe 3 (pipe with two shelves).

the heater. Encounter of the two flows increases the pressure of the gas in the middle part along the pipe, which acts to reduce the flows in each side. The mass fluxes of the two thermal transpiration flows are nearly the same in a slightly rarefied gas, but the resistance to pressure-driven flow is quite different in the two parts. Thus, a one-way flow may be induced. Setting a small windmill near each end of the pipe, we try to detect the one-way flow through the pipe. (See Fig. 1.)

The vacuum chamber is prepared with a glass bell jar put on a steel base, which is usually found in standard laboratories. The chamber is of diameter 250 mm and of height 300 mm, and its pressure  $p$  can be controlled between the atmospheric condition and several pascals by a small oil-sealed rotary vacuum pump (free air displacement: 120 l/min). On the steel base, several electric wire terminals are prepared, so that electricity can be supplied to the heater through them. This is the vacuum chamber where our previous experiments on flows induced by temperature fields (see Sone [12]) were carried out.

We prepare the components, Pipe 1, Pipe 2, Pipe 3, Heater, Guide, and Desk explained below:

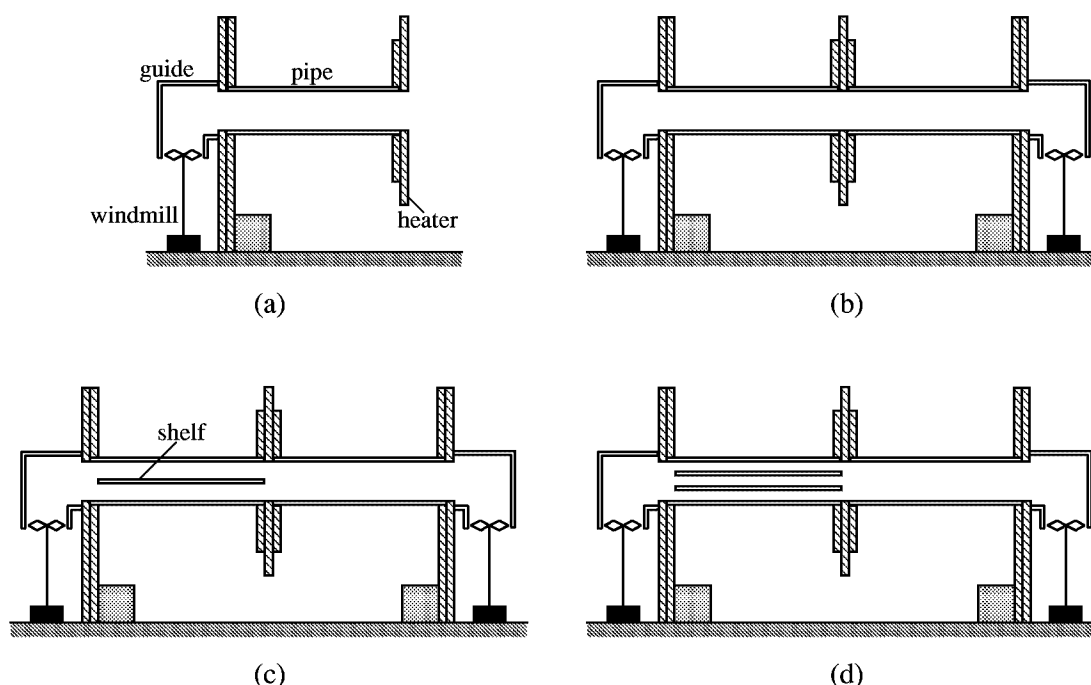
**Pipe 1:** A glass circular straight pipe of inner diameter 11.5 mm, thickness 1.75 mm, and length 65 mm. A copper plate (width 35 mm, height 35 mm, thickness 1 mm) is attached at one end. At this end a Heater is to be attached or another pipe component is to be connected with a Heater being sandwiched. Another copper plate (width 50 mm, height 88 mm, thickness 1 mm) is attached at the other end of the pipe. A Guide is to be attached to this end. The system is to be fixed to Desk with this copper plate (See Fig. 1). This copper plate also works to cool the end at nearly room temperature.

**Pipe 2:** A Pipe 1 the passage of which is separated into equal two parts by putting up a thin glass shelf (0.57 mm thick). Each edge of the shelf is covered with a thin copper plate attached to the main copper plate, which helps to heat or cool the edge. [Fig. 2 (a)]

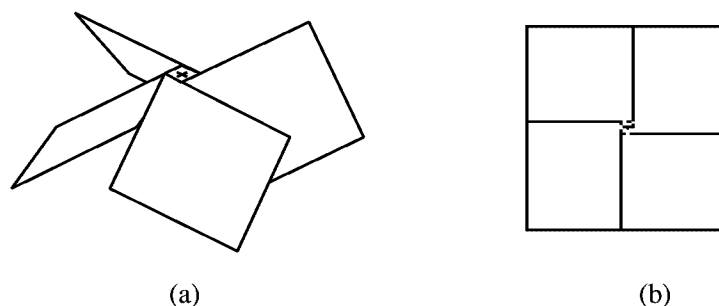
**Pipe 3:** A pipe of Pipe 2 with two shelves, instead of one, to separate the passage into three parts. [Fig. 2 (b)]

**Heater:** A copper plate (width 50 mm, height 50 mm, thickness 1 mm) where a covered Nichrome wire is wound through eight holes on its circumferential area. The plate is to be attached to one of the components or sandwiched between two of the Pipes joined together.

**Guide:** A guide to the windmill as arranged in Fig. 1. The connection to a component is made with a copper plate (width



**FIGURE 3.** The pipe systems. (a) System 1: Single pipe, (b) System 2: Symmetric pipe, (c) System 3: System with a shelf, and (d) System 4: System with two shelves.



**FIGURE 4.** The vanes of the windmill. (a) View of the vanes and (b) The design on a plastic film from which the vanes is made of. In panel (b), the solid line shows a cut and the dashed line shows a crease. A tiny depression, around which the vanes rotate on a needle, is made at the center by pressing a pencil point.

50 mm, height 88 mm, thickness 2 mm) at one of the ends.

Desk: a small steel desk (length 150 mm, width 55 mm, and height 25 mm) on which the pipe system assembled is fixed.

The following four types of pipe system are composed of the above mentioned components:

System 1: The system that consists only of a single Pipe 1. In this system the temperatures at the two ends of the pipe are different. The speed of rotation of the windmill in this system serves as the standard to compare how effectively a one-way flow is induced in a system where two thermal transpirations encounter. [Fig. 3 (a)]

System 2: The system where two of Pipe 1 are joined together with a Heater sandwiched. This system is symmetric with the respect to the Heater, and thus no flow is induced. This serves as a test system. [Fig. 3 (b)]

System 3: The system where Pipe 1 and Pipe 2 are joined together with a Heater sandwiched. [Fig. 3 (c)]

System 4: The system where Pipe 1 and Pipe 3 are joined with a Heater sandwiched. [Fig. 3 (d)]

The temperature gradient along the pipe is induced by the Heater between two components (or at one end in System 1). The temperature is controlled by adjusting the electric current through the wire by a transformer prepared outside the bell jar. A flow induced through a pipe is detected with a small windmill set at the entrance of the guide. The vanes of the

**TABLE 1.** The temperatures ( $^{\circ}\text{C}$ ) at the Heater and the two pipe-ends. The column S is the data at the end of the pipe with a shelf or shelves, and N is those at nonshelf side. In System 1, C is the colder end of the pipe.

$p$ (Pa)	System 1		System 3			System 4		
	C	Heater	S	Heater	N	S	Heater	N
100	27.2	94.5	27.7	95.5	27.3	27.2	94.9	26.6
40	27.4	94.5	27.9	94.5	27.5	27.4	94.4	26.9
20	27.6	94.5	28.1	94.3	27.7	27.6	94.2	27.1
10	27.8	95.0	28.2	94.4	27.8	27.8	94.1	27.3
6.7	28.0	95.4	28.4	94.8	28.0	28.0	94.4	27.5
5	28.2	95.4	28.6	95.0	28.2	28.3	94.9	27.7

windmill are made of a square sheet of a plastic film ( $10 \times 10$  mm), such as transparency film for an overhead projector, with four cuts (Fig. 4). The vanes rotate on a needle around a tiny depression made by pressing a pencil point at their center.

A pipe system is fixed on the Desk with the copper plates, and the windmill is also put on the desk. The whole system is put on the steel base in the vacuum chamber. Large thermal conductivity and heat capacity of the copper plate, steel desk, and steel base work as to keep the two ends of the pipe being of an equal temperature. The Heater is connected with the electric wires on the base, and thermometers (thermocouple testers) are set at the Heater and at the copper plates at the ends of the pipe.

## Results

The measurement is performed in the following way. Prepare the system of a pipe in the vacuum chamber, put the heater on, and wait for a steady state by observing the thermometers. Then, observe a flow by the rotation of the windmill. Decrease the pressure in the chamber and wait for a steady state. Continue the above process and observe the variation of the speed of rotation of the windmill. After various and repeated preliminary experiments, we adopt the following steps: Start at 100 Pa, put on the heater and wait for 90 min. Observe the speed of rotation for 10 min. Then, decrease the pressure and wait for 10 min and then observe the windmill. Repeat the process. The observation at atmospheric condition is done separately.

The temperature of the pipe is kept at  $28^{\circ}\text{C}$  at the ends and at  $95^{\circ}\text{C}$  at the Heater. With these temperatures, the temperatures at intermediate points are, for example, about  $60^{\circ}\text{C}$  at the points at 20 mm from the heater. Their detailed data will be given later.

The results of the observation of the rotation of the windmills are as follows:

- (i) No motion of the windmill (no flow through the pipe) is observed at the atmospheric condition (about  $1.013 \times 10^5$  Pa) for any of the pipe systems.
- (ii) No motion of the windmill is observed at any pressure for any of the pipe systems when the pipe is not heated.
- (iii) No motion of the windmill is observed at any pressure for System 2.
- (iv) Under 100 Pa, steady rotation of the windmill or the windmills is observed in the three systems: Systems 1, 3, and 4. The temperatures at the Heater and at the ends of the pipe during the experiment are shown in Table 1. These data are the average values, and their standard deviations for repeated experiments are about  $1^{\circ}\text{C}$ . Slight increase of temperature is seen at the two ends with time (the experiment is done from higher pressure). The rotation of the windmill shows that a one-way flow is induced from the left to the right in Fig. 3 (a), (c), (d), i.e., from the colder side to the hotter in System 1 and from the pipe with a shelf or shelves to that without it or them in Systems 3 and 4. The speeds of rotation of Systems 1, 3, and 4 between 5 Pa and 100 Pa are listed in Table 2. The simple vanes made of plastic film are not strong enough for repeated experiments. Six series of observation from 100 Pa to 5 Pa for Systems 1, 3, and 4, three and three with the left and the right vanes interchanged, are done, at least, with the same set of two vanes. Thus, some variations are inevitable for such simple vanes. The standard deviations of the data of repeated experiments are shown in parentheses ( $\cdots$ ) in Table 2. These deviations are mainly attributed to the difference of the performance of the vanes. From the examination of the data of the relative speeds of rotation in each experiment from 100 Pa to 5 Pa with respect to the speed at 5 Pa, whose average values are shown in square brackets [ $\cdots$ ] in Table 2, the small differences of the speeds of rotation between 5 Pa and 6.7 Pa are found to be significant. The relation between the speed of rotation and the inverse of pressure, the latter of which is proportional to the mean free path, is plotted in Fig. 5. The speed of rotation increases with  $1/p$ , but the speed of increase decreases. Incidentally, the mean free path of the gas (or air) at 10 Pa is about 0.65

**TABLE 2.** Speed of rotation (per minute) of the windmill I: Main experiment (95°C at the Heater and 28°C at the pipe ends). The column S is the speed of rotation (rpm) of the windmill on shelf side, and N is that of the other. A positive value indicates rotation corresponding to an upward flow, and a negative value to a downward flow. The values in parentheses are the standard deviations of the data of repeated experiments. See the main text for the data in the square brackets.

$p$ (Pa)	System 1	System 3		System 4	
		S	N	S	N
100	23.1 (2.6) [0.044]	10.4 (1.3) [0.093]	−10.5 (1.5) [0.085]	15.7 (1.6) [0.112]	−16.5 (1.8) [0.109]
40	79.0 (4.1) [0.149]	32.3 (2.1) [0.288]	−35.0 (2.0) [0.282]	47.3 (2.5) [0.336]	−49.3 (3.0) [0.324]
20	143.3 (7.2) [0.270]	53.8 (3.1) [0.480]	−57.5 (2.6) [0.463]	76.8 (4.3) [0.546]	−80.3 (5.0) [0.528]
10	297.5 (20.8) [0.560]	89.4 (4.6) [0.797]	−97.4 (6.3) [0.783]	118.5 (7.3) [0.842]	−124.5 (9.0) [0.817]
6.7	450.4 (27.9) [0.849]	107.9 (5.2) [0.963]	−118.7 (7.6) [0.955]	136.5 (8.2) [0.971]	−147.9 (11.9) [0.969]
5	531.2 (40.7) [1]	112.2 (5.9) [1]	−124.5 (10.4) [1]	140.7 (8.8) [1]	−152.7 (13.2) [1]

**TABLE 3.** Speed of rotation of the windmill II: Smaller temperature difference (63°C at the Heater and 27°C at the pipe ends). See the caption of Table 2.

$p$ (Pa)	System 1	System 4	
		S	N
100	10		
40	35	17	−16
20	69.5	31.5	−31
10	164	53	−55

mm, and the Knudsen number based on the diameter of the pipe is about  $5.7 \times 10^{-2}$ . Thus, the Knudsen number from 100 Pa to 5 Pa is from  $5.7 \times 10^{-3}$  to  $1.13 \times 10^{-1}$ .

(v) In the experiments below 10 Pa, a little care must be paid about the condition of pipes. For any pipe system unheated or System 2 heated or unheated, a slow rotation of the windmill showing a downward flow is sometimes observed. This flow persists for a long time (an hours to several hours or more) depending on situations. From repeated preliminary experiments, the flow may be considered to be a gas flow extracted from the contaminated pipe. For a pipe not used for a while, the flow is rather large and persists long, and a pipe continually used, the flow is absent or small (a few rpm) and vanishes rather quickly. Wiping a pipe decreases to vanish the flow.

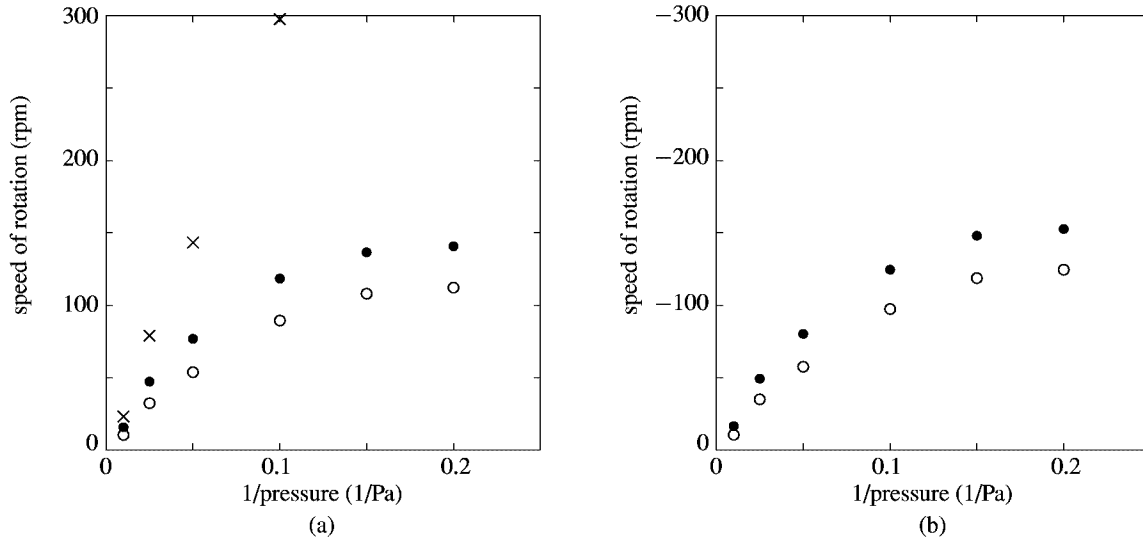
(vi) As a supplementary experiment, the same experiment is done at a different heater temperature. With the Heater at temperature 63 °C and the pipe ends at 27 °C, where the temperature difference is 54% of the main experiment, the speeds of rotation, shown in Table 3, are 43–55% for System 1 and 32–45% for System 4 of those of the main experiment shown in Table 2. These ratios increase with the speed of rotation.

In a simple and visual experiment, it is found that a one-way flow can be induced fairly effectively in a uniform straight pipe with its two ends at an equal temperature and pressure just by putting up a shelf or shelves inside a half part of the pipe. The main processes of the experiment are recorded on a video film.

## DISCUSSION

First we discuss the relation between the speed of rotation of the windmill and the flow speed of the gas under a crude assumption on the properties of system *e.g.*, aerodynamic performance of the vanes of the windmill and the friction at the contact of the needle and vanes. In a steady rotation of the windmill, the aerodynamic force (or moment of force) on the vanes balances with the force (supporting force and moment of friction) on the vanes at the needle. For small speed of rotation of the windmill and small Knudsen number, the force on the vanes may be considered to be proportional to the inverse of pressure with its coefficient linear with respect to the flow speed and the speed of rotation. The balance equation for rotation is  $p(c_1 U - c_2 \omega) = MF$  or  $U = (c_2 \omega + MF/p)/c_1$ , where  $U$  is the flow speed,  $\omega$  is the angular speed of rotation of the windmill, and  $MF$  is the moment of friction. Let  $\omega$  be expressed as  $\Omega/p$ , where  $\Omega$ , according to the experimental data, takes a finite value for small  $1/p$  and decreases with  $1/p$ . Then

$$U = \frac{(c_2 \Omega + MF)}{c_1} \frac{1}{p}, \quad (1)$$



**FIGURE 5.** The speed of rotation of the windmill. (a) The windmill on the left side in Fig. 3 and (b) the windmill on the right side in Fig. 3. The symbol  $\times$  indicates the data for System 1, the symbol  $\circ$  for System 3, and the symbol  $\bullet$  is for System 4. The positive value corresponds to a upward flow and the negative to a downward flow.

where the friction  $MF$  may be considered to be a constant, and with increase of the Knudsen number (or  $1/p$ ), the deviations of  $c_1$  and  $c_2$  from the original values become appreciable. The flow speed  $U$  increases linearly with  $\Omega$ . It may be noted here that the effect of gas rarefaction (or  $1/p$ ) on  $\Omega$  in the measurements appears in the two different ways: the effect on the windmill ( $c_1, c_2, MF$ ), which determines the response of  $U$ , is common to the systems, but the effect on  $U$ , on which  $\Omega$  depends, appears most magnified in System 4 because of the largest effective Knudsen number in the presence of two shelves.

Now we will discuss the mechanism of the one-way flow. Thermal transpiration of a slightly rarefied gas through a straight pipe of an arbitrary cross section induced by the temperature gradient along the pipe is very simply obtained with the aid of asymptotic theory (see Sone [13,14] and their references). That is, the flow at the leading term is uniform over the cross section with magnitude of the thermal creep slip (see Sone [12] and its references) with a Knudsen-layer correction in a close neighborhood of the pipe surface. The mass flux  $M_T$  through the pipe is simply given by

$$M_T = \frac{c_T l p S}{T_w \sqrt{RT_w}} \frac{dT_w}{dx}, \quad (2)$$

where  $T_w$ ,  $dT_w/dx$ ,  $p$ ,  $l$ ,  $S$ ,  $R$ , and  $c_T$  are, respectively, the temperature of the pipe, its gradient along the pipe (assumed to be a constant for simplicity), the pressure of the gas, the mean free path of the gas molecules, the cross sectional area of the pipe, the specific gas constant, and a constant depending on the molecular model. The mass flux is proportional to the cross sectional area. On the other hand, the flow induced by a pressure gradient along the pipe is the Poiseuille flow, and the leading term of the mass flux for a slightly rarefied gas is given by

$$M_P = -\frac{c_P p S^2}{\mu R T_w} \frac{dp}{dx}, \quad (3)$$

where  $dp/dx$ ,  $\mu$ , and  $c_P$  are, respectively, the pressure gradient of the gas, its viscosity, and a constant depending on the shape of the cross section of the pipe. (The viscosity is related to the mean free path  $l$  as  $\mu/p = f(T/T_0)(2RT_0)^{1/2}l$ , where  $T$  is the temperature of the gas,  $T_0$  is a constant with dimension of temperature, and  $f$  is a function depending on molecular models. For a hard-sphere gas,  $f = 0.56277$  [12].) The mass flux is in the opposite direction of the pressure gradient and proportional to the square of the cross sectional area.

In Systems 2, 3, or 4, on each side of the heater in the pipe, a thermal transpiration flow is induced toward the Heater. Encounter of the two flows increases the pressure of the gas in the middle part along the pipe, which acts to reduce the flows on each side. Here, a pressure gradient as well as the temperature gradient is induced in the gas. We will estimate the flow induced in this system with the aid of Eqs. (2) and (3). For simplicity, consider a pipe of rectangular cross

section, where its half part is divided into  $m \times m$  equal passages by shelves and the other half is without any shelf. At the position of the Heater, the temperature and pressure are continuous. Thus, let  $dT_w/dx$  and  $dp/dx$  be, respectively, the temperature and pressure gradients on the side of the pipe with shelves (say, Part S), and then those on the side of the pipe without them (say, Part N) are  $-dT_w/dx$  and  $-dp/dx$ . The mass flux  $M_S$  in Part S is given by

$$M_S = AS \frac{dT_w}{dx} - m^2 B \left( \frac{S}{m^2} \right)^2 \frac{dp}{dx}, \quad (4)$$

where  $A = c_T l p T_w^{-1} (RT_w)^{-1/2}$  and  $B = c_P p / \mu RT_w$ . It is noted that  $dp/dx$  as well as  $dT_w/dx$  is a constant, because the mass flux is constant along System S. The mass flux  $M_N$  in Part N is given by

$$M_N = -AS \frac{dT_w}{dx} + BS^2 \frac{dp}{dx}. \quad (5)$$

The mass flux due to thermal transpiration is independent of the existence of shelves if their thickness is neglected because of its linear dependence on  $S$ . The two fluxes should be equal ( $M_S = M_N = M$ ;  $M$ : the mass flux through the pipe) because of mass conservation. Therefore, the mass flux through the pipe  $M$  is given by

$$M = \left( \frac{m^2 - 1}{m^2 + 1} \right) AS \frac{dT_w}{dx}. \quad (6)$$

The  $(m^2 - 1)/(m^2 + 1)$  in Eq. (6) is a factor expressing the reduction by joining the inverse-temperature-gradient parts to keep the two ends of the pipe at an equal temperature. For more shelves, more flux is induced, but the factor fairly quickly approaches unity (e.g., it is 15/17 for  $m = 4$ ). Too many shelves do not increase the flux effectively. Incidentally if some averaged  $c_P$  is used, the formula applies to a pipe of an arbitrary cross section divided into  $n$  passages instead of  $m^2$ , though the averaged  $c_P$  depends on  $n$ . The factor  $(n - 1)/(n + 1)$  [i.e., 1/3 for System 3 or 1/2 for System 4] gives a rough estimate of the experimental data especially above 10 Pa.

The above estimate, however, is approximate: The discussion is based on the formulas for an infinitely long pipe with small temperature and pressure gradients, and the end effect is neglected, especially the discussion on the behavior in the neighborhood of the connection of Part S and Part N; the passages separated by shelves are not equal cross sections; the formulas of mass fluxes are the leading contributions for a slightly rarefied gas. (A software on Windows 95 by which the velocity profiles and mass fluxes of thermal transpiration and Poiseuille flow through a channel between two parallel plates or through a circular pipe at an arbitrary Knudsen number are given in a second is available [15].) Thus, the result (6) does not give an accurate answer of the problem, but the effect of separation into thinner passages can be well understood. The one-way flow observed in the experiment is explained by the difference of dependence on the cross sectional area of the pipe between the thermal transpiration and Poiseuille flow. The thermal transpiration is of equal strength in the pipe with a shelf or shelves and in the pipe without it, but the pressure-driven flow is weaker in the former. The transpiration exceeds the pressure-driven flow in the pipe with a shelf or shelves, but it goes in the other way round in the pipe without a shelf. As the result, a one-way flow is induced.

Incidentally, it is noted that the estimate by the above rough discussion agrees with the mathematical conclusion that no one-way flow is induced in a straight uniform pipe without a shelf heated at an arbitrary position along the pipe. Let Part N is  $\alpha$  times longer than Part S. Then,  $M_S$  is given by Eq. (4) with  $m = 1$  and  $M_N$  by Eq. (5) with  $dT_w/dx$  and  $dp/dx$  replaced by  $\alpha^{-1}dT_w/dx$  and  $\alpha^{-1}dp/dx$ . From the condition  $M_S = M_N$ , we have  $M_S = M_N = 0$ . Here, the momentum induced by thermal transpiration in Part N is the same magnitude as and the opposite direction to that in Part S (smaller velocity and larger mass in Part N for  $\alpha > 1$ ), and two flows cancel after interaction.

To summarize, a one-way flow can easily be induced through a uniform straight pipe kept its two ends at an equal condition (an equal temperature and pressure) by subdividing the passage of the pipe partially. The effect of subdivision fairly quickly saturates to the maximum value (or the value of the case with different end conditions where no counter flow is induced), and thus subdivision into too many passages is ineffective. Its mechanism is explained by the difference of dependence on the cross sectional area of the passage between thermal transpiration and Poiseuille flow.

Incidentally, we will estimate the effect of the shelf on the pumping effect of the pipe, that is, to close the pipe at the ends to block the flow and to estimate the pressure difference induced at the two ends of the pipe. When the mass flux is blocked ( $M_S = 0$  and  $M_N = 0$ ), the pressure gradient  $dp/dx$  induced in each part of the pipe is, according to Eqs. (4) and (5) [note that the pressure gradient along Part N in Eq. (5) was  $-dp/dx$ ], given by

$$\frac{dp}{dx} = \frac{A}{B} \frac{m^2}{S} \frac{dT_w}{dx} \quad (\text{in the pipe with a shelf or shelves}), \quad (7)$$

$$\frac{dp}{dx} = -\frac{A}{B} \frac{1}{S} \frac{dT_w}{dx} \quad (\text{in the pipe without a shelf}). \quad (8)$$

Thus, the pressure difference  $\Delta p$  at two ends of the pipe is

$$\frac{\Delta p}{L} = \frac{A}{2B} \frac{(m^2 - 1)}{S} \frac{dT_w}{dx}, \quad (9)$$

where  $L$  is the length of the pipe. The pressure difference increases indefinitely with the number of the shelves. The effect of the shelf appears quite differently in the one-way flow and in the pumping effect. The flat plane shelves parallel to the pipe axis and their negligible thickness do not affect the thermal transpiration flux, but the pressure driven flow is reduced to vanish by their resistance with their number. Thus, the pressure difference required to block the thermal transpiration in the part with shelves increases indefinitely. It may be noted that in a porous media where the passage is so narrow that the free molecular approximation is applicable, the relation  $p/\sqrt{T} = \text{const}$  applies to any shape of passage when the flow is blocked. [16] Combination of the formula with Eq. (8) gives the formula of pressure difference for a system of a porous media and a pipe. This does not give an infinite pressure difference.

Finally, there are various kinds of flows induced by temperature fields, in addition to thermal transpiration, which will have applications to flow control. Their comprehensive information can be obtained in review articles as Kogan *et al.* [17], Kogan [18], and Sone [12].

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